Enhanced FP2 Examination (MEI)

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Rules

Allowed time: 1 hour 30 minutes.

Answer all questions.

This is an enhanced examination paper for the further mathematics module *Further Pure Mathematics 2*. It is meant to be a significantly more challenging examination of general concepts specified by MEI. It is fully doable for students with knowledge of standard A-level mathematics and further mathematics.

1 If $\mathbf{A}$ is diagonisable and $\mathbf{A} = \mathbf{PDP}^{-1}$, show that $\mathbf{A}^n = \mathbf{PD}^n\mathbf{P}^{-1}$.

2 Show that $\operatorname{sech}(2x) = \left(\frac{\tanh^2 x - 1}{\tanh^2 x + 1}\right)^2$.

3 Find $\int \frac{1}{\sqrt{8-2x-x^2}}\, dx$.

4 **(a)** Find the cube roots of $3 - i$.

(b) Explain why the sum of the nth roots of a complex number is zero.

5 Show that $\sin(\arctan \frac{x}{y}) = \frac{x}{\sqrt{x^2+y^2}}$.

6 Evaluate the sum

$$\sum_{r=5}^{25} \frac{2}{(r-3)(r-1)}$$

7 Find the two lines of points invariant under $\begin{pmatrix} -1 & 2 \\ -6 & 6 \end{pmatrix}$.

8 If the Maclaurin series for a differentiable function $f(x)$ is

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(k)}(0)}{k!}x^k + \ldots$$

show that if $n \in \mathbb{R}$ the Maclaurin series for $f(nx)$ is

$$f(nx) = f(0) + f'(0)\frac{x}{n} + \frac{f''(0)}{2}\left(\frac{x}{n}\right)^2 + \ldots + \frac{f^{(k)}(0)}{k!}\left(\frac{x}{n}\right)^k + \ldots$$