Mechanics 1 (M1)

Enhanced Practice Examination

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Time Allowed: 1 hour 30 minutes | **Total Marks:** 75 marks

Take acceleration due to gravity $g = 9.8\ m\ s^{-2}$. Answer all questions. Show all necessary working.

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1. [Kinematics with Variable Acceleration] [9 marks]

A particle P moves in a straight line along the x-axis. At time $t \geq 0$ seconds, its acceleration a m s^{-2} is given by:

$$a(t) = 6t - 18$$

At $t = 0$, P passes through the origin O with initial velocity $u = 24$ m s^{-1}.

(a) Find an expression for the velocity $v(t)$ of P at time t. [3]

(b) Find the two times at which the particle is instantaneously at rest. [2]

(c) Calculate the total distance travelled by the particle in the first 5 seconds of motion. [4]

2. [Connected Particles over a Smooth Pulley] [11 marks]

Two particles A and B, of mass 3 kg and 5 kg respectively, are connected by a light inextensible string passing over a small, smooth, fixed pulley. The system is released from rest with both particles hanging vertically at a height of 2 m above horizontal ground.

(a) Form equations of motion for particles A and B, and hence calculate the acceleration of the system and the tension in the string. [5]

(b) Find the speed of B when it strikes the ground. [3]

(c) Assuming B does not rebound, find the maximum height above the ground reached by particle A. [3]

3. [Dynamics on an Inclined Rough Plane] [11 marks]

A package of mass 4 kg is held at rest on a rough plane inclined at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$. The coefficient of friction between the package and the plane is $\mu = 0.25$.

(a) Show that if released from rest with no external forces, the package will accelerate down the plane, and calculate its acceleration. [5]

(b) A horizontal force of magnitude P newtons is applied to the package to prevent it from slipping down the plane. Determine the minimum value of P required for equilibrium. [6]

4. [Statics: Coplanar Equilibrium of a Suspended Body] [8 marks]

A small light ring R is threaded onto a light inextensible string of length 14 m. The ends of the string are attached to two fixed points A and B at the same horizontal level, where $AB = 10$ m. A load of mass 12 kg is suspended from the ring.

(a) Explain why the tension in both sections of the string AR and BR must be equal. [2]

(b) Find the vertical depth of the ring below the level of AB in equilibrium. [3]

(c) Calculate the tension in the string. [3]

5. [Moments: Non-Uniform Rigid Rod] [12 marks]
A non-uniform rod AB of length 6 m and mass 40 kg rests horizontally in equilibrium on two smooth supports placed at C and D, where $AC = 1$ m and $DB = 1$ m. When a mass of 20 kg is placed at end A, the rod is on the point of tilting about C.

(a) Find the distance of the center of mass of the rod from end A. [5]

(b) The mass at A is removed. A vertical downward force of magnitude F is now applied at end B to make the rod on the point of tilting about D. Find the value of F. [4]

(c) When no external loads are applied, find the reaction forces exerted on the rod by the supports at C and D. [3]

6. [2D Vectors and Projectiles] [10 marks]
A particle of mass 0.5 kg moves in a horizontal plane. At time t seconds, its velocity vector is given by:

$$\mathbf{v}(t) = (4 - 3t)\mathbf{i} + (2t - 5)\mathbf{j} \quad (\text{m s}^{-1})$$

(a) Find the magnitude and direction of the resultant force $\mathbf{F}$ acting on the particle. [4]

(b) Find the time t at which the particle is moving in the direction of the vector $2\mathbf{i} - \mathbf{j}$. [3]

(c) Find the kinetic energy of the particle when $t = 2$ s. [3]

7. [Momentum and Direct Collisions] [14 marks]
Two smooth spheres P and Q of equal radius and masses m and $2m$ respectively are moving along the same straight line on a smooth horizontal floor. Sphere P moves with speed $4u$ and sphere Q moves with speed u in the same direction, such that P collides directly with Q. The coefficient of restitution between P and Q is $e = \frac{1}{2}$.

(a) Show that immediately after the collision, the speed of P is u and the speed of Q is $\frac{5}{2}u$. [6]

(b) Find the magnitude of the impulse exerted by P on Q during the impact in terms of m and u. [3]

(c) Calculate the loss in total kinetic energy of the system due to the impact, expressed as a fraction of the initial total kinetic energy. [5]

END OF EXAMINATION